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LETTER TO THE EDITOR

Dynamics of the two-dimensional electron gas in the lowest Landau level: a continuum elasticity approach

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Abstract. We develop a bosonization scheme for the collective dynamics of a spinless two-dimensional electron gas (2DEG) in the lowest Landau level. The system is treated as a continuous elastic medium, and quantum commutation relations are imposed between orthogonal components of the elastic displacement field. The theory provides a unified description of bulk and edge excitations of compressible and incompressible phases and allows the calculation of the electronic tunnelling current, yielding results which are in good agreement with recent experiments at the edge and in the bulk of the 2DEG.

The two-dimensional electron gas (2DEG) in a high magnetic field such that all the electrons reside in the lowest Landau level (LLL) provides an extraordinarily rich laboratory to test ideas about the 'non-Fermi-liquid' behaviour of strongly correlated electrons [1]. The incompressible quantum liquid state introduced by Laughlin [2] to explain the fractional quantum Hall effect [3] at rational filling factors; the pseudogap in the tunnelling density of states observed at generic (compressible) densities [4]; the likely formation of a Wigner crystal at low density [5]; and the realization [6] that the edges of this system behave as a 'chiral Luttinger liquid' [7–9] at essentially all filling factors, are prime examples of this richness.

A theoretical treatment of the system is notoriously difficult, since the electron–electron interaction cannot be treated perturbatively. Several nonperturbative approaches have been developed, such as the variational theory of Laughlin [2], extended to the collective dynamics by Girvin *et al* [10], and, more recently, the boson or fermion Chern–Simons theories [11–14], which replace the original electrons with composite particles for which the mean-field treatment is a reasonable starting point. All these theories are designed to work best at some special, rational filling factors, such as $1/3, 1/2$ etc. Another fruitful line of attack has been the hydrodynamic treatment of density fluctuations: this allows bosonization of the edge excitations of the 2DEG [15–17], but fails to reproduce the spectrum of collective excitations in the bulk.

The purpose of this letter is to outline a novel approach to the linear dynamics of a *general* uniform or nonuniform distribution of electrons in the LLL. Unlike previous theories [10–18], the present one treats bulk and edge excitations of compressible and incompressible states on an equal footing. Under physically motivated assumptions, we shall argue that the 2DEG is dynamically equivalent to a set of noninteracting bosons, and we shall give a prescription for expressing the electron tunnelling operator in terms of these bosons [19].

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The basic idea of our approach is suggested by the similarity between the local structure of the 2DEG in the LLL and that of a Wigner crystal (this fact was exploited by Johansson and Kinaret [20] to set up an 'independent-boson model' for the tunnelling spectral function; see also reference [18] for a derivation of their 'independent-boson model' from diagrammatic many-body theory). In the absence of real crystalline order, we propose to treat the electrons, at a macroscopic level, as a continuous elastic medium characterized by an equilibrium density $\rho_0(\vec{r})$, and by a local elastic displacement field $\vec{u}(\vec{r},t)$, such that the volume element which is located at $\vec{r}$ in the ground state is found at $\vec{r}+\vec{u}(\vec{r})$ in the excited state considered. The time-dependent density is, to first order in $\vec{u}$, $\rho(\vec{r},t)=\rho_0(\vec{r})-\vec{\nabla}\cdot[\rho_0(\vec{r})\vec{u}(\vec{r},t)]\equiv\rho_0(\vec{r})+\delta\rho(\vec{r},t)$. We ignore the spin in this letter. Under the assumption that both the equilibrium density and the displacement are *slowly varying* on the scale of the magnetic length $l=(\hbar c/eB)^{1/2}$ (where B is the magnetic field) the effective long-wavelength Hamiltonian in the LLL is then

$$H=\frac{1}{2}\int \mathrm{d}\vec{r}\int \mathrm{d}\vec{r}'\ \delta\rho(\vec{r})\left(\frac{e^2}{|\vec{r}-\vec{r}'|}-\tilde{\chi}^{-1}(\vec{r},\vec{r}')\right)\delta\rho(\vec{r}') + \int \mathrm{d}\vec{r}\ \mu\left[\rho_0(\vec{r})\right]\sum_{\alpha,\beta}\left[s_{\alpha\beta}(\vec{r})-\frac{1}{2}\delta_{\alpha\beta}\vec{\nabla}\cdot\vec{u}(\vec{r})\right]^2 \tag{1}$$

where $\tilde{\chi}^{-1}(\vec{r},\vec{r}')$ is the inverse of the *proper* static density–density response function [21], μ is the shear modulus (discussed below), $s_{\alpha\beta}\equiv[\partial u_\alpha(\vec{r})/\partial r_\beta+\partial u_\beta(\vec{r})/\partial r_\alpha]/2$ is the strain tensor, and α,β are cartesian indices. Equation (1) generalizes the Hamiltonian of classical elasticity theory by [22] taking into account the long range of the Coulomb interaction, and of $\tilde{\chi}(\vec{r},\vec{r}')$ in the incompressible states [10].

The second term of equation (1) is an essential and novel feature of the present approach. The physical idea is that the strongly correlated electron fluid is very viscous: hence, the time τ during which a density fluctuation relaxes to equilibrium is much longer than the characteristic timescale of the linear dynamics. It follows that, over a range of frequencies $1/\tau\ll\omega\lesssim e^2/\hbar l$, the system should behave like an elastic solid, characterized by a nonvanishing shear modulus and negligible dissipation. Only on a longer timescale $\omega\lesssim 1/\tau$ does viscous damping become important and the shear modulus drop to zero. In this letter we shall assume that $1/\tau\to 0$, for long-wavelength fluctuations, sufficiently rapidly to justify the use of the Hamiltonian (1) at essentially all relevant frequencies. We believe this assumption to be fully justified for the incompressible states, since these are known to be free of dissipation. It should also be qualitatively correct for general compressible states, to the extent that the local structure of correlations resembles that of a Wigner crystal [23].

The algebra of the displacement operators u_x,u_y, in the LLL, is deduced from the canonical quantization condition for the hydrodynamical momentum and displacement fields $[p_\alpha(\vec{r}),u_\beta(\vec{r}')]=-\mathrm{i}\delta(\vec{r}-\vec{r}')\delta_{\alpha\beta}$, by projecting out the higher Landau levels, i.e. averaging over the fast cyclotron motion. This leads to

$$[u_\alpha(\vec{r}),u_\beta(\vec{r}')]=-\mathrm{i}\epsilon_{\alpha\beta}\delta(\vec{r}-\vec{r}')\,l^2/\rho_0(\vec{r}) \tag{2}$$

where $\epsilon_{\alpha\beta}$ is the two-dimensional Levi–Civita tensor.

Bosonization of (1) is accomplished by using

$$u_\alpha(\vec{r})=\frac{l}{\rho_0^{1/2}(\vec{r})}\sum_{n>0}\left[b_n g^*_{n\alpha}(\vec{r})+b_n^\dagger g_{n\alpha}(\vec{r})\right] \tag{3}$$

where b_n and $b_n^\dagger$ are boson operators, satisfying the usual algebra $[b_n,b_{n'}^\dagger]=\delta_{nn'}$ etc. The functions $g_{n\alpha}(\vec{r})$ ($g^*_{n\alpha}(\vec{r})$), with $n>0$, are the positive- (negative-) frequency solutions of

the eigenvalue problem

$$\int H_{\alpha\beta}(\vec{r},\vec{r}')g_{n\beta}(\vec{r}')\ \mathrm{d}\vec{r}' = \omega_n g_{n\alpha}(\vec{r}) \tag{4}$$

which is equivalent to the classical equation of motion in the limit of infinite magnetic field. (We adopt the convention of summing over repeated cartesian indices.) The operator $H_{\alpha\beta}$ is given by

$$\begin{aligned} H_{\alpha\beta}(\vec{r},\vec{r}') = &-\mathrm{i}l^2\rho_0^{1/2}(\vec{r})\epsilon_{\alpha\gamma}\left[\partial_\gamma\partial'_\beta\chi^{-1}(\vec{r},\vec{r}')\right]\rho_0^{1/2}(\vec{r}') \\ &+ \mathrm{i}\frac{l^2}{\rho_0(\vec{r})}\epsilon_{\alpha\gamma}\left[\partial_\beta\partial'_\gamma - \partial_\gamma\partial'_\beta + \delta_{\gamma\beta}\partial_\eta\partial'_\eta\right]\left[\delta(\vec{r}-\vec{r}')\mu\right] \end{aligned} \tag{5}$$

where $\chi^{-1}(\vec{r},\vec{r}') \equiv \tilde{\chi}^{-1}(\vec{r},\vec{r}') - e^2/|\vec{r}-\vec{r}'|$, and ∂_α and ∂'_β denote derivatives with respect to r_α and r'_β respectively. It is Hermitian with respect to the scalar product

$$(f,g) \equiv \int f^*_\alpha(\vec{r})\mathrm{i}\epsilon_{\alpha\beta}g_\beta(\vec{r})\ \mathrm{d}\vec{r}$$

and its eigenvalues are therefore real. For each eigenfunction g_n with positive eigenvalue ω_n, there is a complex conjugate one $g^*_n \equiv g_{-n}$ with negative eigenvalue $-\omega_n$. These eigenfunctions can be chosen to satisfy orthonormality relations $(g_n, g_m) = \mathrm{sgn}(n)\delta_{nm}$ (n and m can now have either sign) [24]. The 'completeness relation' has the form

$$\sum_{n>0}[g_{n\alpha}(\vec{r})g^*_{n\beta}(\vec{r}') - g^*_{n\alpha}(\vec{r})g_{n\beta}(\vec{r}')] = \mathrm{i}\epsilon_{\alpha\beta}\delta(\vec{r}-\vec{r}'). \tag{6}$$

These equations guarantee that the commutation relation (2) is satisfied. Then, substitution of equation (3) into equation (1) yields the Hamiltonian in the desired form

$$H = \sum_{n>0}(b^\dagger_n b_n + 1/2)\omega_n.$$

Our next step is to construct the electron tunnelling operator $\psi(\vec{R})$. This operator is expected to create a Gaussian density perturbation near $\vec{R}$ and no change in the vorticity $\rho_v(\vec{r}) \equiv -\epsilon_{\alpha\beta}\partial_\alpha[\rho_0(\vec{r})u_\beta(\vec{r})]$ [25]:

$$[\psi(\vec{R}), \rho(\vec{r})] = \psi(\vec{R})\exp[-|\vec{r}-\vec{R}|^2/2l^2]/2\pi l^2 \tag{7}$$

and $[\psi(\vec{R}), \rho_v(\vec{r})] = 0$. The solution has the form

$$\psi(\vec{R}) = C_{\vec{R}}\exp\left[-\sum_{n>0}\left(\frac{M^*_n(\vec{R})}{\omega_n}b_n - \frac{M_n(\vec{R})}{\omega_n}b^\dagger_n\right)\right] \tag{8}$$

where the 'electron–phonon' matrix elements $M_n(\vec{R})$ are written as

$$M_n(\vec{R}) = \int \exp[-|\vec{r}-\vec{R}|^2/2l^2]\tilde{M}_n(\vec{r})\ \mathrm{d}\vec{r}/2\pi l^2$$

and

$$\frac{\tilde{M}_n(\vec{r})}{\omega_n} = \frac{\mathrm{i}}{2\pi}\int \epsilon_{\alpha\beta}\frac{g_{n\alpha}(\vec{r}')}{\rho_0^{1/2}(\vec{r}')l}\ \partial_\beta \log|\vec{r}-\vec{r}'|\ \mathrm{d}\vec{r}' \tag{9}$$

and $C_{\vec{R}}$ is an operator which commutes with the bosons and decreases the total particle number by one [26]. A serious deficiency of the tunnelling operators is that they fail to satisfy fermionic anticommutation rules at different points: this defect should not have major consequences for the calculation of the tunnelling current, as long as the tunnelling electron remains essentially distinguishable from the pre-existing ones.

The calculation of the local Green's function $G(\vec{R},t) = -\mathrm{i}\langle T\psi(\vec{R},t)\psi^{\dagger}(\vec{R},0)\rangle$ can be carried out by standard techniques [27, 20]. Here we simply report the zero-temperature result for the integral equation connecting the electronic spectral function $A(\vec{R},\omega)$ (the Fourier transform of $G(\vec{R},t)$) to the collective excitation spectrum:

$$\omega A(\vec{R},\omega) = \int_0^{\omega} g(\vec{R},\Omega)\, A(\vec{R},\omega-\Omega)\, \mathrm{d}\Omega \tag{10}$$

where $\omega > 0$ and

$$g(\vec{R},\Omega) = \frac{1}{\Omega}\sum_{n>0} |M_n(\vec{R})|^2 \delta(\Omega-\omega_n) \tag{11}$$

is closely related to the local dynamical structure factor of the liquid. The above equations constitute a complete scheme for the calculation of single-particle and collective properties of a general distribution of electrons in the LLL. We now discuss some specific examples.

(1) *The uniform 2DEG*. Let ρ_0 be the uniform density. The normalized eigenfunctions of equation (4) are

$$g_{\vec{q}L}(\vec{r}) = \mathrm{i}ql(\mu/2\omega_q\rho_0)^{1/2}\mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{r}} \tag{12}$$

and

$$g_{\vec{q}T}(\vec{r}) = (ql)^{-1}\left(\omega_q\rho_0/2\mu\right)^{1/2}\mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{r}} \tag{13}$$

where

$$\omega_q = \mu^{1/2}\left(v(q) + \frac{K(q)+\mu}{\rho_0^2}\right)^{1/2}(ql)^2 \tag{14}$$

is the frequency of the mode at wavevector $\vec{q}$, $v(q)$ is the Fourier transform of the electron–electron interaction, $K(q) = -\rho_0^2\tilde{\chi}^{-1}(q)$ is the q-dependent bulk modulus, and the labels L (longitudinal) and T (transverse) refer to the components parallel and perpendicular to $\vec{q}$. The electron–phonon matrix element of equation (9) has the form

$$M_q(\vec{r}) = \left[v(q) + \frac{K_{\vec{r}}(q)+\mu}{\rho_0^2}\right]\mathrm{e}^{-q^2l^2/2}\rho_q(\vec{r}) \tag{15}$$

where $\rho_q \equiv -\mathrm{i}ql\rho_0^{1/2}g_{qL}(\vec{r})$ and

$$K_{\vec{r}}(q) \equiv -\rho_0^2\int \tilde{\chi}^{-1}(\vec{r}-\vec{r}')\exp[\mathrm{i}\vec{q}\cdot(\vec{r}'-\vec{r})]\,\mathrm{d}\vec{r}'.$$

(The reason for the apparently unnecessary subscript $\vec{r}$ will become clear below.)

We now distinguish two cases:

(i) *The compressible case.* $\lim_{q\to 0} K(q) = K_c$ is finite, and small compared to the Coulomb repulsion $\rho_0^2 v(q)$. The value of μ can be approximated by that of a classical Wigner crystal, namely, $\mu \sim \mu_c = 0.09775\rho_0(e^2/l)\nu^{1/2}$ [28]. The long-wavelength modes have frequencies $\omega_q \propto q^{3/2}$ for Coulomb-like interaction ($v(q) = 2\pi e^2/q$) and $\omega_q \propto q^2$ for short-range interaction ($v(q) = e^2 d$). The low-frequency behaviour (where 'zero' frequency is the chemical potential) is found from equations (10), (11) to be $A(\omega) \propto \omega^{-5/4}\mathrm{e}^{-\gamma/\omega^{1/2}}$ for Coulomb interaction and $A(\omega) \propto \omega^{(1+(2.7\nu)^{1/2}d/l)^{1/2}/4\nu-1}$ for short-range interaction. Complete numerical results are shown in figure 1. They are in good agreement with previous calculations [20, 18] as well as with experimental data [4].

(ii) *The incompressible case.* Within the single-pole approximation [10] $K(q\to 0) \simeq \Delta\rho_0/2\alpha q^4 l^4$, where Δ is the collective excitation gap at $q = 0$, and $\alpha = (1-\nu)/8\nu$ is

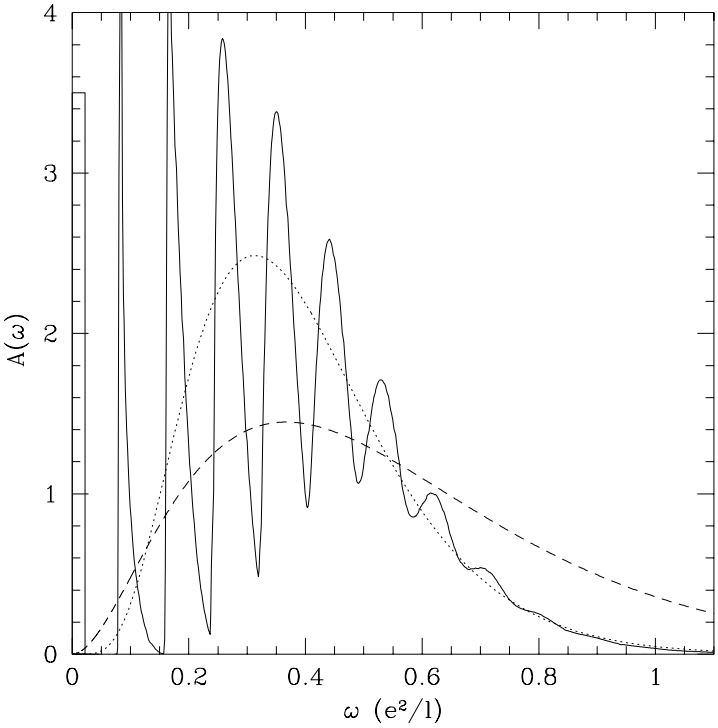

Figure 1. The spectral function for a compressible liquid at $\nu = 0.3$ with short-range interactions ($d = 13l$, dashed curve), with long-range interactions (dotted curve) and for an incompressible liquid at $\nu = 1/3$ with short-range interactions (same d, full curve, the δ-function peak at $\omega = 0$ contains around 8% of the spectral strength). In all cases we used $K_c = 0$.

derived from the Laughlin wavefunction at filling factors $\nu \equiv 2\pi l^2 \rho_0 = 1/\text{odd integer}$ [2, 10]. Using equation (14) we find at long wavelength $\omega_q = (\Delta\mu/2\alpha\rho_0)^{1/2}$ independently of q. This formula can be used to deduce the value of μ if $\Delta = \lim_{q\to 0} \omega_q$ is known. Alternatively, one can substitute $\mu = \mu_c$ from the classical Wigner crystal and obtain $\Delta_\nu = 0.391\nu^{3/2}(1-\nu)^{-1}e^2/l$, which gives $\Delta_{1/3} = 0.11e^2/l$, $\Delta_{1/5} = 0.044e^2/l$ and $\Delta_{1/7} = 0.025e^2/l$. These results are in good agreement with variational estimates [10, 29], with the exception of the first one which is almost 30% lower than the variational one, but compares favourably with exact-diagonalization studies [30].

The calculation of the spectral function is more subtle. Straightforward application of the linear response formulae (10)–(15) is incorrect, because the addition (or removal) of charge at point $\vec{R}$ creates a compressible region in the middle of the liquid, changing the *topology* of the incompressible region from simply to doubly connected. The change in topology can be taken into account in the following manner. We stipulate that the bulk modulus kernel $-\rho_0^2\tilde{\chi}^{-1}(\vec{r},\vec{r}')$ has the form characteristic of an incompressible liquid (namely, $|\vec{r}-\vec{r}'|^2\log|\vec{r}-\vec{r}'|$) when both $\vec{r}$ and $\vec{r}'$ are within the incompressible region, but it is given by the local form $K_c\delta(\vec{r}-\vec{r}')$ when either $\vec{r}$ or $\vec{r}'$ are within the compressible 'core' of the excitation. Because the size of the core region is microscopic, its presence does not significantly affect the frequencies of the long-wavelength modes. On the other hand, in the 'electron–phonon' matrix element, given by equation (15), we must use $K_{\vec{r}}(q) \simeq K_c \ll \rho_0^2 v(q)$, because $\vec{r}$ is inside the core region. Within this scheme, the

calculation of the spectral function can be straightforwardly carried out. In figure 1, we plot the results for $\nu = 1/3$. We have used q-dependent K and μ in order to fit accurately the collective mode dispersion and structure factor [10] at finite q. The essential difference between this and the compressible case is the appearance of a δ-function peak at $\omega = 0$, which now corresponds to the energy for the addition of three quasiparticles or quasiholes [31]. The strength of the δ-function is

$$Z = \exp\left[-\sum_q |M_q|^2/\omega_q^2\right]$$

which would have vanished in the compressible case, and *does not vanish here because of the gap*. The presence of the peak reflects the ability of the incompressible liquid to accommodate the incoming electron as a localized topological defect of the initial state, without creating collective excitations. The incoherent part of the spectral function, at higher frequencies, corresponds to the creation of additional collective excitations.

(2) *Edge dynamics*. The case of a smooth compressible edge has been treated in previous publications [16]. Here we focus on the case of a *sharp* edge, which is directly relevant to the interpretation of recent *lateral* tunnelling experiments by Chang *et al* and Grayson *et al* [6], and which has recently attracted widespread theoretical interest [32]. Let us consider a straight edge along the y-axis, and let $eE\hat{x}$ be the gradient of the confinement potential at the edge. The equilibrium density ρ_0 is assumed to be uniform for $x < 0$, and zero for $x > 0$. We believe this to be approximately true, because the Coulomb energy tends to suppress charge inhomogeneities on a sufficiently macroscopic scale. We note that this 'weak-incompressibility' assumption has nothing to do with the 'strong incompressibility' responsible for the quantum Hall effect. The presence of the edge electric field breaks the rotational symmetry of the Hamiltonian (1), and can be taken into account with the additional term [33]

$$H^{\text{edge}} = e\int \rho(\vec{r})\vec{u}(\vec{r})\cdot\vec{E}\ \mathrm{d}\vec{r}. \tag{16}$$

With this term included into the eigenvalue problem (4), we obtain a set of solutions, bound to the edge, which satisfy the conditions $\vec{\nabla}\cdot\vec{g} = 0$ and $\vec{\nabla}\times\vec{g} = 0$ for $x < 0$. Neglecting the corrections arising from the long range of the Coulomb interaction, they obey the equation of motion

$$-\mathrm{i}\omega g_\alpha(\vec{r}) = v\epsilon_{\alpha\beta}\,\partial_\beta g_x(\vec{r}) \tag{17}$$

where $v = cE/B$ is the classical drift velocity. The orthonormal solutions are (for $x < 0$)

$$\vec{g}_q(\vec{r}) = q^{1/2}\mathrm{e}^{qx-\mathrm{i}qy}(\hat{x} - \mathrm{i}\hat{y}) \tag{18}$$

where $q > 0$ is the wavenumber along the edge, and the eigenvalues are $\omega_q = vq$. They are analogous to gravity waves on the surface of a liquid. Because there is neither density change nor shear strain in the interior of the system, these solutions do not depend on the bulk elastic constants. The use of elasticity theory is justified at small q since the displacement field $\vec{u}$ is slowly varying.

The effective edge dynamics can be derived from the full dynamics by projecting the latter onto the subspace spanned by the edge-wave solutions of equation (18). The chief physical assumption is that any electron tunnelling *laterally* into the system can and will be accommodated via the creation of edge-wave deformations. Within this subspace, we can define an 'edge density' operator

$$\rho^{\text{edge}}(y) \equiv \int \delta\bar{\rho}(x, y)\ \mathrm{d}x = \rho_0\bar{u}_x(0, y)$$

where the bar denotes projection onto the edge-wave subspace, i.e., for instance,

$$\bar{u}_x(0, y) \equiv (l/\rho_0^{1/2}) \sum_{q>0} q^{1/2}[b_q e^{iqy} + b_q^\dagger e^{-iqy}].$$

It is easy to verify that the edge density satisfies the standard Kac–Moody algebra [8] $[\rho_q^{\text{edge}}, \rho_{-q'}^{\text{edge}}] = (\nu q/2\pi)\delta_{qq'}$ where ν is the bulk filling factor. Thus, we have deduced the dynamics of the chiral Luttinger liquid at the edge from a projection of the canonical dynamics of displacement fields in the bulk. The edge tunnelling operator, satisfying the condition $[\psi(y), \rho^{\text{edge}}(y')] = \psi(y)\delta(y - y')$ in the edge-wave subspace, can be written in a form similar to equation (8), with $\vec{R} \to y$, the sum running over the edge modes, and the matrix element $M_q(y) = v(2\pi q/\nu)^{1/2} e^{-iqy}$. Use of equations (10) and (11) then leads to the conclusion that the tunnelling current must vanish, at low bias V, as $V^{1/\nu}$. This result is in good agreement with the experimental findings, and the present derivation would explain why the tunnelling exponent does not seem to depend on whether the bulk exhibits the fractional quantum Hall effect or not. However, both the projection onto the edge-wave subspace, and the use of the nonfermionic operator ψ to describe the tunnelling, lack a rigorous justification.

In summary, we have developed a magneto-elastic bosonization scheme for the long-wavelength dynamics of the 2DEG in the LLL. Our results show that this scheme can provide a unified description of different physical effects in the bulk and at the edge of this system.

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[23] The compressible state at $\nu = 1/2$ is anomalous from the present point of view, being the accumulation point of an infinite series of incompressible states [13]. Its description requires the inclusion of q-dependent viscosity terms.
[24] The extra sign is caused by the property of the scalar product $(f, f) = -(f^*, f^*)$ together with the definition $g_{-n} = g_n^*$. The fact that the positive-frequency eigenfunctions ($n > 0$) are the ones with positive norm follows from the stability of the ground state.
[25] For example, in Laughlin's thought experiment [2] to generate a quasiparticle, the addition of a magnetic flux at the origin causes a radial displacement of all the electrons, with vanishing curl.
[26] Because $c_{\vec{R}}(t) = c_{\vec{R}} \mathrm{e}^{-\mathrm{i}\mu t}$ the only effect of $c_{\vec{R}}$ is to shift the zero of the energy to the chemical potential μ.
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[33] The derivation of the edge Hamiltonian (16) makes use of the 'weak-incompressibility' condition $\vec{\nabla} \cdot \vec{u} = 0$ and neglects the (small) shear modulus of the edge. Details will be given elsewhere.